Conditional Non-Bias of Geostatistical Simulation for Estimation of Recoverable Reserves

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Abstract

Conditional bias is an infamous problem with estimation methods including kriging. Changing estimation parameters will mitigate, but not remove, conditional bias. The conditional bias of kriging is well understood; however, there is widespread confusion in the literature and among practicing geostatisticians regarding the conditional bias of geostatistical simulation. There is no conditional bias of simulation when the simulation results are used correctly. The correct use of simulation for recoverable reserves estimation is to (1) generate multiple realizations conditional to all available data at a small scale, (2) linearly average all realizations to the chosen block size and (3) calculate the probability of each block being ore and the ore grade of each block. The "probability of ore" and the "ore grade" are conditionally non-biased.

Introduction

The problems of conditional bias are notorious. Although there have been significant developments in geostatistics, conditional bias is still a concern of practical ore reserve estimation. Most mines base recoverable reserves on some type of estimates. Such estimates are often conditionally biased. Attempts to improve estimation algorithms have met with some success, but the problems of conditional bias persist.

Conditional bias occurs when the expected value of the true grade (Z_V) conditional on the estimated grade ($Z^*_V = z$) is not equal to the estimated grade, that is:

$$E\{Z_V \mid Z^*_V = z\} \neq z \tag{1}$$

where the V is symbolic of some volume of estimation, for example, a selective mining unit (SMU). Conditional bias is almost always present due to the smoothing effect of all linear estimation procedures, including kriging, in the presence of sample data that are relatively widely spaced. The true grade is typically less than the estimated grade when the estimated grade is high and the true grade is typically greater than the estimated grade when the estimated grade is low.

Conditional non-bias occurs when the expected value of the true grade conditional on the estimated grade is equal to the estimated grade. In practice, this requires that material believed to have an average grade of 1 g/t will, in fact, have an average grade of 1 g/t, and that material supposed to have an average grade of 10 g/t will, in fact, have an average grade of 10 g/t. Conditional non-bias is much more stringent than global non-bias. Often, in practice, local accuracy is sacrificed for global accuracy, that is, conditional bias is accepted in turn for global non-bias.

A small example has been constructed to illustrate conditional bias (see Figure 1). Consider mineralization with a lognormal histogram with a mean of 1.0 and a variance of 4.0. The variogram has a relative nugget effect of 20% and an isotropic range of 40 m. Consider four samples on a regular 20m grid to estimate a central 10 m square block in a high and low-grade case. This is a favorable estimation scheme since the nugget effect is relatively low and the range is large relative to the sample spacing. Nevertheless, there is conditional bias, that is, the kriged grade is too high in the high-grade case and too low in the low-grade case.

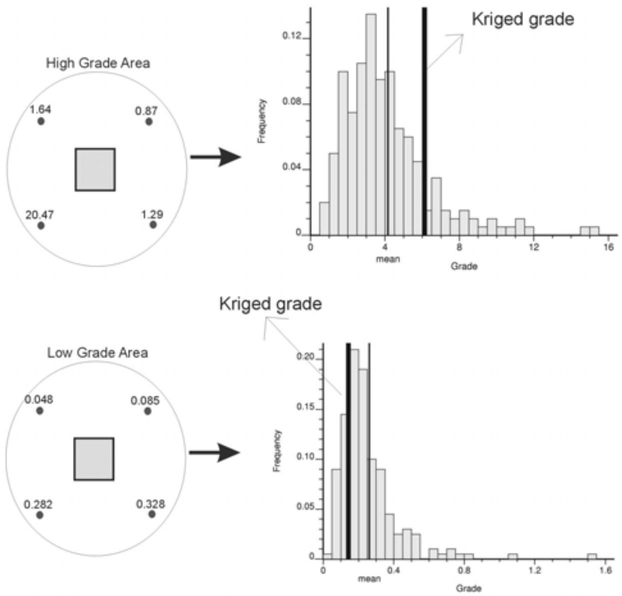

Figure 1. The sketches to the left show the data configuration and a central block being estimated (10 m on a side) in a high and low-grade case. The histograms show the distributions of true grades conditional to the 20m spaced sample data. The heavy vertical line is the kriged grade and the light vertical line is the mean true grade. The kriged grade is too high in the high-grade case and too low in the low-grade case.

Mine planning is often based on estimates calculated from blasthole or drillhole data. Adequate follow-up with validation exercises on these estimates is important. In practice, however, ore/waste selection is based on relatively widely spaced sample data and limited access to true grades at the same support. This often results in smoothed and/or conditionally biased selective mining unit (SMU) estimates and, therefore, misleading SMU selection, recoverable reserve estimates and profit profiles.

There are many flavors of kriging. Uniform Conditioning (UC), Direct Conditioning (DC), Disjunctive Kriging (DK), the MultiGaussian approach (MG), the Bi-Gaussian approach (BG) and Median Indicator Kriging (MIK) are a few, see Guibal and Remacre (1984) or Marcotte and David (1985). The classical application of kriging to mine planning is discussed in David (1977) and Journel and Huijbregts (1978). The relationship between conditional biases, smoothing and various search routines is well understood and documented, see Krige and Assibey-Bonsu (1999d, 2000), Krige (1994, 1996, 1999, 2000) or Isaaks and Davis (1999) for more recent references.

There are two schools of thought related to the conditional bias and smoothing of ordinary kriging for mine planning:

1. The "conditional bias of block estimates is always wrong" school championed by D.G. Krige, W. Assibey-Bonsu and coworkers. Here, one never accepts block estimates known to be wrong in expected value. Large search routines retaining many conditioning data are implemented to minimize uncertainty and conditional bias. The price is block estimates that are smooth and near the mean.
2. The "let's get recoverable reserves right" school championed by various other practitioners. The idea here is to anticipate the dispersion variance of the true block grades. Fewer samples are used in the kriging plan to increase the variability of the block estimates. The price here is block estimates that are conditionally biased.

We see the relative merits of both schools; however, there is no reasons to pay with too smooth estimates or conditional bias. We would rather pay with the increased computational and professional time to implement simulation correctly. Creating "probability of ore" and "ore grade" SMU estimates derived from multiple geostatistical simulations eliminates conditional bias, corrects smoothing, and accounts for uncertainty. This paper demonstrates how valid recoverable reserve estimates can be derived from geostatistical simulation.

The essential idea of using geostatistical simulation for the estimation of recoverable reserves is to (1) apply geostatistical simulation to quantify the uncertainty in block grades and account for smoothing, (2) calculate the "probability of ore" ($\mathbf{P}_{ORE}$) for every SMU as the proportion of simulated block grades above cutoff, and (3) calculate the "ore grade" ($\mathbf{Z}_{ORE}$) for every SMU as the average of the simulated block grades above cutoff.

Calculating probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ estimates for all the mining blocks allows for improved recoverable reserves for mine planning; however, these estimates must be properly validated and shown to be conditionally non-biased for various cutoff grades. The entire procedure can be largely automated so that the mining engineer/geologist gets maps of the $\mathbf{P}_{ORE}$ and $\mathbf{Z}_{ORE}$ SMU estimates and their corresponding cross validation plots. The $\mathbf{P}_{ORE}$ and $\mathbf{Z}_{ORE}$ block estimates are shown to be conditionally non-biased.

There are numerous advantages to basing mine plans and recoverable reserves on probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ SMU estimates. Such estimates are not conditionally biased, have the right dispersion variance and account for inherent block grade uncertainties. The uncertainty involved in selection is conveniently represented by $\mathbf{P}_{ORE}$ maps and the expected ore grade within any subsection of the orebody is straightforward to calculate with $\mathbf{Z}_{ORE}$ maps. The increased CPU demand of this approach is not an issue.

The recommended methodology is presented with all necessary detail. A simple example will illustrate the procedure.

Methodology

The language of probability is a well-established way to express uncertainty. Multiple geostatistical realizations of the grades at a fine scale, conditioned by all available data, are constructed with Gaussian or indicator simulation techniques to express our uncertainty in the grades:

$$\{z^{(l)}(\mathbf{u}_q),\ q = 1,\dots,F,\ l = 1,\dots,L\} \tag{2}$$

where the locations $\mathbf{u}_q$, $q = 1,\dots,F$ represent a finely gridded version of the orebody. There should be 10 or more "fine" blocks per SMU of volume V and the number of realizations $l = 1,\dots,L$ should be sufficient to reflect uncertainty and avoid decision making based on unrepresentative stochastic features (20-100 are considered sufficient).

These small-scale realizations are linearly block averaged to the mining size V to obtain a new set of L realizations at the SMU resolution:

$$\{z_V^{(l)}(\mathbf{u}_j),\ j = 1,\dots,N,\ l = 1,\dots,L\} \tag{3}$$

The probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ for the N SMU locations are calculated from the L realizations while applying a cutoff grade z_C. An indicator transform is defined for every mining block, for every realization:

$$\mathbf{i}(z_V^{(l)}(\mathbf{u}_j); z_C) = \begin{cases} 1, & if \quad z_V^{(l)}(\mathbf{u}_j) > z_C \\ 0, & otherwise \end{cases} \qquad for \quad j = 1,\dots,N\ ;\ l = 1,\dots,L \tag{4}$$

The probability of ore at each location is calculated:

$$\mathbf{P}_{ORE}(\mathbf{u}_j) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{i}(z_V{}^{(l)}(\mathbf{u}_j); z_C) \qquad for \quad j = 1,...,N; l = 1,...,L \tag{5}$$

The ore grade at each location is calculated:

$$\mathbf{Z}_{ORE}(\mathbf{u}_j) = \frac{\sum_{l=1}^{L}\mathbf{i}(z_V{}^{(l)}(\mathbf{u}_j); z_C)\cdot z_V{}^{(l)}(\mathbf{u}_j)}{\sum_{l=1}^{L}\mathbf{i}(z_V{}^{(l)}(\mathbf{u}_j); z_C)} \qquad for \quad j = 1,...,N; l = 1,...,L \tag{6}$$

The probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ SMU estimates are useful for mine planning. The ore grade $\mathbf{Z}_{ORE}$, ore tonnes $\mathbf{T}_{ORE}$ and waste tonnes $\mathbf{T}_{WASTE}$ for a particular subset of *N'* mining blocks for a particular stage of mining is calculated:

$$\begin{aligned} \mathbf{Z}_{ORE} &= \frac{\sum_{j=1}^{N'}\mathbf{P}_{ORE}(\mathbf{u}_j)\cdot\mathbf{Z}_{ORE}(\mathbf{u}_j)}{\sum_{j=1}^{N'}\mathbf{P}_{ORE}(\mathbf{u}_j)} \qquad for \quad j = 1,...,N' \\ \mathbf{T}_{ORE} &= T_V\cdot\sum_{j=1}^{N'}\mathbf{P}_{ORE}(\mathbf{u}_j) \qquad for \quad j = 1,...,N' \\ \mathbf{T}_{WASTE} &= T_V\cdot\sum_{j=1}^{N'}[1-\mathbf{P}_{ORE}(\mathbf{u}_j)] \qquad for \quad j = 1,...,N' \end{aligned} \tag{7}$$

where T_V is the tonnes of a full block *V*. Of course, the specific gravity could be modeled geostatistically on a by-rock-type basis.

We must now demonstrate that the $\mathbf{P}_{ORE}$ and $\mathbf{Z}_{ORE}$ SMU estimates are conditionally unbiased, that is, we must show that:

$$\begin{aligned} &E\{\mathbf{P}_{ORE}, z_C \mid \mathbf{P}^*_{ORE}, z_C = p\} = p \qquad \forall\ p \in [0,1] \\ &E\{\mathbf{Z}_{ORE}, z_C \mid \mathbf{Z}^*_{ORE}, z_C = z\} = z \qquad \forall\ z \end{aligned} \tag{8}$$

These cannot be proven in all generality since the true distribution of grades will never exactly follow our random function (RF) models; however, we could indeed prove these conditionally non-bias conditions if the true grades and the simulated grades follow the same RF model, that is, the uncertainty in the true grades is modeled correctly by the conditional distribution functions $F_V^*(\mathbf{u}_j; z_C)$ sampled by the simulation. In practice, if our simulated realizations reflect the true grades and we have sufficient data to estimate all parameters (variogram, histogram, etc) with no error then $F_V(\mathbf{u}_j; z_C) \approx F_V^*(\mathbf{u}_j; z_C)$ for all locations $\mathbf{u}_j$, all volumes *V* and all cutoff grades z_C. Truncated statistics such as $\mathbf{P}_{ORE}$ and $\mathbf{Z}_{ORE}$ are the same if the distributions are the same, and satisfy relation (8).

The conditional non-bias of $\mathbf{P}_{ORE}$ and $\mathbf{Z}_{ORE}$ estimates can also be demonstrated with extensive data or by a simulation exercise. We could check the true SMU grades coincident with a subset of *N* SMUs with predicted probability of ore *p* +/- Δp. There should be $\frac{n}{N} \approx p$ of that subset actually ore, where *n* is the number of true SMU grades above cutoff. For example, if there are 250 SMUs with predicted probability of ore 0.10 +/- 0.025 then 25 of those SMUs should truly be above cutoff, that is, $\frac{n \; = 25}{N \; = 250} \approx p = 0.10$.

And the true ore grade should be equal to the estimated ore grade $\mathbf{Z}_{ORE}$ at each SMU.

The preceding approach is simple. The $\mathbf{P}_{ORE}$ estimates are simply the proportions of grade realizations above cutoff, and the averages of those grades (identified to be above cutoff) are the $\mathbf{Z}_{ORE}$ estimates. Checking the conditional non-bias of these estimates is simple and consistent with the theory of cross validation.

An Example

The grades in this example are synthetic, but were chosen to mimic practical mineralization. Application to real data is straightforward; however, access to sufficient true grades is limited. The proposed approach will work for any particular grade or variable *z*. The parameters used in Krige (2000) and Assibey-Bonsu are used for easy comparison.

The exhaustive true grade model is created by unconditional simulation using the “sgsim” program from GSLIB. A grid of 120 by 120 by 120 (1,728,000) values spaced 1m apart with a lognormal distribution with a mean grade of 1.0 and a variance of 1.7 is built. The variogram has a 41% relative nugget effect with ranges of 19m in the horizontal and 35m in the vertical.

A 20m spaced sample data set is now extracted. This 20m grid of samples (216 values) will be used as exploration data for subsequent kriging and simulation.

Checking the conditional bias of probability of ore and ore grade estimates requires knowing the true grades at the chosen SMU support. An SMU size *V* of 10 by 10 by 10m is chosen and a true grade model is calculated by linearly block averaging the fine-scale true grade model. The variance of the re-blocked values (the true dispersion variance) is 0.52.

Ordinary kriging (conditioned to the 20m grid of exploration samples) with 4 and 24 maximum conditioning data is performed with the “kt3d” program from GSLIB (see Figure 2). With 4 conditioning data, the true underlying block variance is well reproduced (0.55 obtained vs. 0.52 sought) and the slope of the regression line is 0.53; however, with 24 maximum data, the variance is severely underestimated (0.20 vs. 0.52 sought) and the slope of the regression line is 0.88. As expected, the conditional bias when using 4 maximum data is much more severe than when using 24 maximum data.

One hundred fine-scale geostatistical realizations, conditioned by the 20m grid of exploration data are simulated. These realizations are then block averaged to the SMU scale.

The results of the probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ SMU estimates are summarized in Figure 3. We assumed a base case cutoff grade of 0.5. The high-grade areas have high probabilities of being ore and high expected ore grades while the low-grade areas have low probabilities of being ore and low expected ore grades. The minimum of the probability of ore estimates is 0.17 and the expected ore grades are all greater than the 0.5 cutoff.

To show that the probability of ore $\mathbf{P}_{ORE}$ and ore grade $\mathbf{Z}_{ORE}$ SMU estimates are conditionally unbiased, six symmetric probability intervals are chosen. The centers of these probability intervals are the estimated probabilities. The proportions of true SMU grades above cutoff within each probability interval are the true probabilities. The average grade of ore is calculated. In Figure 4, a cross plot of the true probabilities of ore versus the estimated probabilities of ore and the true ore grades versus the estimated ore grades are shown. There is perfect correlation between the true and estimated probabilities of ore (the first dot was omitted because there was no probability of ore estimate within this symmetric probability interval) and good agreement between the mean of true ore grades and estimated ore grades.

We explored cutoff grades at 0.5, 1.0, and 1.5 (below, at and above the mean). The results are shown in Figure 5. In all cases, excellent correlation between the true and estimated probabilities of ore exists. The same sensitivity analysis is done for the ore grade estimates. The agreement of true and estimated ore grades also persists.

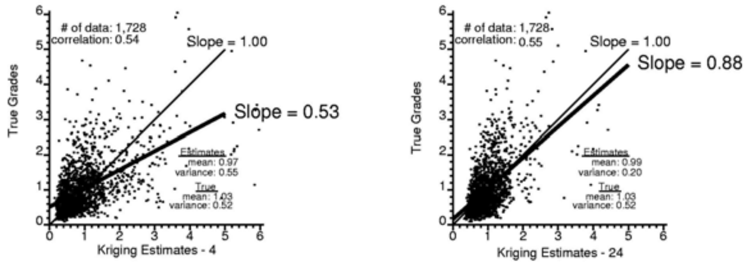

Figure 2. Cross-plots of the true SMU grades vs. ordinary kriging estimates with 4 and 24 maximum conditioning data. There is severe conditional bias and good variance reproduction when kriging with 4 conditioning data, and mild conditional bias and poor variance reproduction when kriging with 24 maximum data.

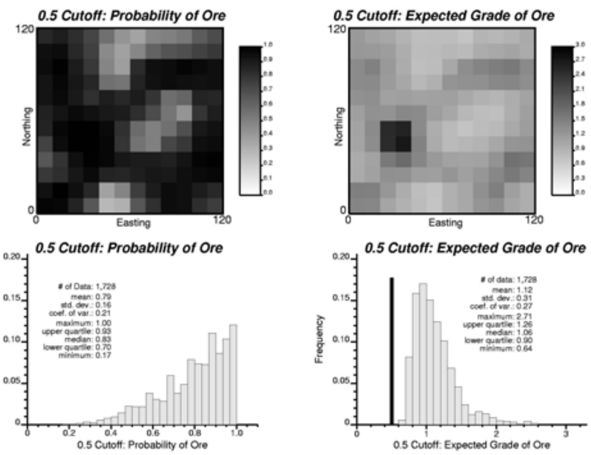

Figure 3. Probability of ore and ore grade estimates using a 0.5 base-case cutoff grade at the central X-Y slice (top). The histogram of all 1,728 probabilities of ore and ore grade estimates are shown. The minimum of the probability estimates is 0.17 and all of the estimated ore grades are greater than the 0.5 cutoff.

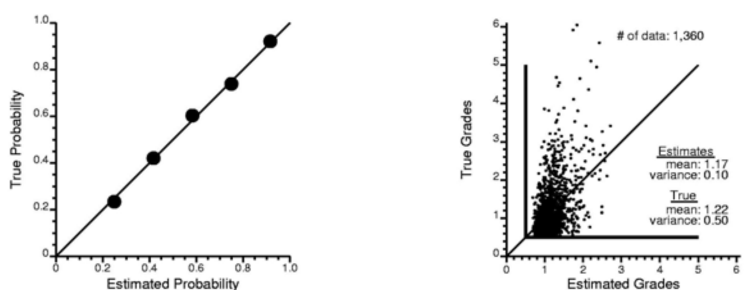

Figure 4. The cross-plot of true versus estimated probability of ore and true vs. estimated ore grade for the 0.5 base-case cutoff. The probabilities are excellently correlated and the mean ore grades are in good agreement.

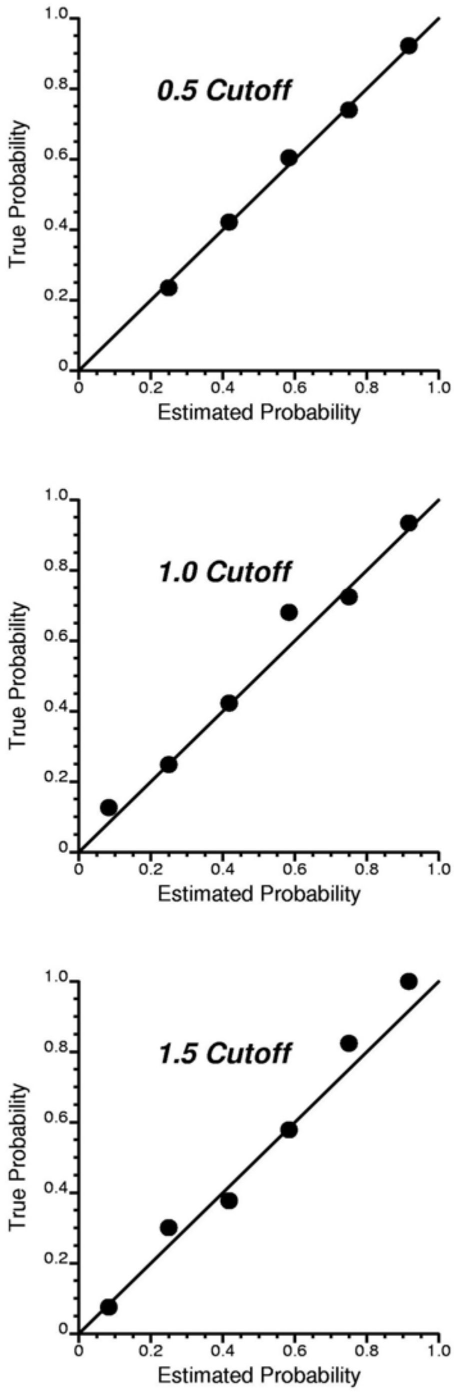

Figure 5. **The conditional bias of probability of ore estimates for cutoff grades at 0.5, 1.0 and 1.5 (below, at and above the mean). Excellent correlation persists insensitive to the cutoff grade.**

Discussion

The results are not surprising. Proper application of simulation, as we have described it, will always work when the simulated realizations make use of the same parameters (histogram, variogram, etc) as the true grades. Nevertheless, it is valuable to illustrate the correct way to use simulation in mine planning. The example would be more convincing with real data; however, it is uncommon to have sufficient true grades for proper testing.

This example has confirmed conventional wisdom, that is, ordinary kriging with a limited search routine results in good variance reproduction at the expense of conditional bias and ordinary kriging with a large search reduces conditional bias at the expense of smoothing. These are important because practitioners do not have the background, resources, or management support to switch to simulation using the methodology we have documented. The practitioner will, therefore, still be faced with the hard choice of improved global estimation of recoverable reserves or estimates with no conditional bias.

Conclusion

Basing mine plans on probability of ore and ore grade estimates calculated from multiple geostatistical realizations effectively solves the longstanding problem of conditional bias. Geostatistical simulation, the theory of block averaging, the calculation of probability of ore estimates and ore grade estimates are consistent with recoverable reserves estimation for mine planning. Implementation is straightforward and the procedure can be run on low-level PC's available at virtually every mine site.

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